Compositionality in Inferential-Role Semantics
(Early Draft)
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Among the more popular theories of conceptual content is inferential-role semantics (henceforth IRS). Roughly, IRS claims that a concept’s content is determined by the role it plays in reasoning. Although this view has had no shortage of supporters (e.g., Field, 1977; Block, 1986; Harman, 1987; Pollock, 1989; Brandom, 2000), defenders of the view have yet to answer one of its most significant objections. The objection is that IRS is incapable of dealing with the compositional nature of concepts. In this paper I propose that a solution to this problem can be found in theories of defeasible inference.

1 Inferential-Role Semantics

IRS claims that the key to a concept’s content is how it is inferentially connected to other concepts. Let’s say that whenever two concepts, \(C_1\) and \(C_2\), can be tokened in the premise and conclusion of an inference, there is an inferential connection between \(C_1\) and \(C_2\), (represented \(C_1 \circ C_2\)). For example, from “The glass contains water” one might infer “The glass contains a liquid.” We could then say that the concept WATER is inferentially connected to the concept LIQUID. What IRS says is that a concept’s content is the set of all of its inferential connections.

Some concepts are compositional. For example, the concept WATERMELON-ART is composed of the concepts WATERMELON and ART. So, a good theory of concepts needs to explain how an agent can start out with concepts like WATERMELON and ART and get WATERMELON-ART. Feodor has spent a good deal of time demonstrating that developing this explanation is going to be a non-trivial enterprise. However, Fodor is particularly dubious of the idea that IRS can account for compositionality. Like Fodor, I think a theory of concepts needs to explain compositionality. Like Fodor, I think the explanation is non-trivial. Unlike Fodor, I think IRS is up to the task.

The basic problem to solve is this: The content of some concepts is (substantially) based upon the content of the concepts which compose them. WATERMELON-ART has the content it does, at least in part, because WATERMELON and ART have the content they do.
For IRS this means that the inferential role of \textsc{watermelon-art} needs to be composed of the inferential roles of \textsc{watermelon} and \textsc{art} respectively. So far so good. But the inferential role of \textsc{watermelon-art}, doesn’t look like the simple addition of the inferential role of \textsc{watermelon} with the inferential role of \textsc{art}. More precisely, the set of inferential connections for \textsc{watermelon-art} is not the union of the sets of inferential connections for \textsc{watermelon} and \textsc{art}. For instance, \textsc{watermelon} has an inferential connection to \textsc{edible} while \textsc{art} has an inferential connection to \textsc{inedible}.

In order to adequately address the compositionality problem IRS needs a principled method of generating the inferential role of a compositional concept from its constituent concepts. In Section 4 I will suggest that with a few tweaks a theory of defeasible reasoning can provide such a method. First, however, it will be worth while to look at why a theory of compositionality is important and why IRS might have difficulty in this regard.

2 Arguments for the compositionality of concepts

Aside from the intuitively compelling fact that \textsc{watermelon-art} looks like it gets some of its content from \textsc{watermelon} and some from \textsc{art}, there are also some quite good arguments that a substantial number of any agent’s concepts must be compositional. These arguments stem from the fact that concepts are productive and systematic.

To say that concepts are productive is to say that a mind can have an unbounded number of content distinct concepts.(Cummins, 1996) But since minds (ours at least) begin with only finite resources, there must be an upper bound on the number of concepts they begin with. The conclusion which is typically drawn from this is that a mind begins with a finite set of primitive concepts which form the constituents of more complex concepts. Compositionality just is getting new concepts out of old ones, so a theory of compositionality would explain the productivity of concepts.\footnote{This seems like the most plausible answer to productivity, however, there is an alternative view. The compositionality explanation of productivity takes it for granted that we have a fixed representational system. Starting out with a fixed finite representational system, the only way to attain an unbounded number of possible representations is to combine preexisting representations. But if the system itself can change, then it is possible to have an unbounded number of possible representations by having an unbounded number of possible systems. This is roughly the idea behind connectionist views of representation. In a connectionist}
To say that concepts are systematic is to say that a mind that can entertain certain concepts can necessarily entertain other concepts with a related, yet distinct, content. For example, anyone that has the concepts UNMARRIED-MALE and FEMALE should be able to form the concept UNMARRIED-FEMALE. Likewise, anyone that has the concepts BROWN-COW and BLACK should be able to form the concept BLACK-COW. An obvious explanation of the systematicity of concepts is that concepts like BROWN-COW and BLACK-COW are compositional concepts whose constituents are the concepts COW, BROWN, and BLACK.

It seems pretty clear that concepts are productive and systematic. Intuitively, it also seems clear that concepts like WATERMELON-ART are getting some of their content from WATERMELON and some from ART. So, a theory of concepts which can explain compositionality will be a theory which can explain productivity, systematicity, and some fairly strong intuitions as well.

3 Fodor and the composition of inferential roles

There seems to be good reason to accept that concepts are compositional. If IRS is the right theory of conceptual content, then IRS should be compatible with the compositional nature of some concepts. Fodor, however, has argued that inferential roles cannot compose. Here is a rather lengthy quote from Fodor on the subject:

Compositionality says that, whatever content is, constituents must yield theirs to their hosts and hosts must derive theirs from their constituents. . . . Now, complying with the first half of this constraint is easy for [IRS] since BROWN contributes to BROWN COW not only its content-constitutive inferences (whichever those may be), but every inference that holds of brown things in general. . . . But the second half of the compositionality constraint is tricky for a [IRS]. If nothing can belong to the content of BROWN COW except what it inherits either from BROWN or from COW, then the content of BROWN COW can’t be its whole inferential role. For, of course, all sorts of inferences can hold of brown cows (not qua brown or qua cows but) simply as such. (Fodor, 1998, pp. 106-107, author’s italics)

The primary problem in this argument is that Fodor takes an overly simplistic view network a new representation is added by adjusting the weights between the nodes that already exist. The addition of the new representation changes how everything is represented. So rather than adding a representation to a fixed system, a connectionist network changes the system.
of the rules of composition. What determines the content of a compositional concept is the content of its constituents and the rules of composition. Fodor’s objection assumes that the only rule of composition available to IRS is that the inferential role of the complex concept is the union of the inferential connections of its constituent concepts. However, as we will see in Section 4 significantly more complex rules are available to IRS. These rules can both add and remove inferential connections from those of the constituent concepts.

4 Composition through reasoning

The solution I propose is that the theory of inference one adopts can help in developing a theory of concepts. The basic problem in finding the inferential role of a compositional concept is determining which inferential connections to keep from the constituent concepts and which new connections, if any, to make. My suggestion is that most of this work can be straightforwardly accomplished by a theory of defeasible reasoning.

For expository purposes it will be useful to go through an example before diving into the details. This example will gloss over a couple of important points but it serves to get the basic idea. To form the inferential role of WATERMELON-ART an agent begins by supposing there is some object $o$ to which WATERMELON and ART can be applied. Following the inferential connections of WATERMELON the agent might defeasibly conclude EDIBLE $o$, GREEN-EXTERIOR $o$, and RED-INTERIOR $o$, among other things. Following the inferential connections of ART an agent might concluded INEDIBLE $o$ and BEAUTIFUL $o$. At this point the agent has the conflicting beliefs, EDIBLE $o$ and INEDIBLE $o$. The resolution of this conflict is where the theory of defeasible reasoning takes over. Precisely, how the conflict gets resolved will depend on the theory of reasoning we adopt.

After all the conflicts are resolved (or at least the pressing ones) the agent is left with a set of beliefs about $o$ based on the supposition that WATERMELON $o$ and ART $o$. This set of beliefs forms the basis of the inferential connections for WATERMELON-ART. In particular the inferential role consists of connections between WATERMELON-ART and those concepts employed in the set of beliefs that have been formed about $o$. Included in this set will be the inferential connections WATERMELON-ART ◦ BEAUTIFUL, WATERMELON-ART ◦ RED-INTERIOR, etc.
That covers the basic idea. Unfortunately, there are a half dozen things wrong with this simplistic version. Now might be a good time to dive into the details. In doing so I will be taking on board a simplistic version of the theory of defeasible reasoning outlined in Pollock (1995). However, most of what follows would remain constant through other theories of defeasible reasoning such as Bayesianism.

**If you are in a technical mood please read on to the next bold face comment. Otherwise skip to section 5.**

To begin we need a brief characterization of defeasible inference. First, there are two kinds of inference: forwards and backwards. In forwards inference an agent moves from a belief in \( P \) to a belief in \( Q \). This is the sort of inference that everyone is familiar with. However, it is just as common to engage in backwards inference. In backwards inference an agent begins with an interest in believing \( Q \) and then develops an interest in believing \( P \) for the purpose of concluding \( Q \). For example, if an agent wants to know whether tomorrow is Friday, then she will develop an interest in knowing whether today is Thursday, for the purpose of concluding that tomorrow is Friday. Second, in defeasible inference some reasons are better than others. Compare reading that today is Friday in the newspaper and having an absent minded professor say it is Friday. Both count as reasons to believe that today is Friday, but the first is a stronger reason than the other. This is reflected by the *reason strength* of the inference.

This brief sketch of defeasible reasoning leads to a more detailed account of the nature of inferential connections. Let \( A \) and \( B \) be concepts and \( A \circ B \) symbolize that \( A \) is inferentially connected to \( B \). \( A \)'s inferential connections to \( B \) will come in two forms. In the first case an agent can make an inference from the application of \( A \) to the application of \( B \). In the second case an agent can make an inference from the application of \( B \) to the application of \( A \). The former I will call a forward connection and symbolize as \( A \rightarrow \circ B \). The later I will call a backward connection and symbolize as \( A \leftarrow \circ B \). We can now define \( A \circ B \) as follows:

**Forward Inferential Connection:** A concept \( A \) has a forward inferential connection to a concept \( B \), \( (A \rightarrow \circ B) \), for an agent \( s \) if the following condition holds:

If \( s \) applies \( A \) to \( o \), then \( s \) is disposed believe \( B o \) on the basis of \( A o \).

**Backward Inferential Connection:** A concept \( A \) has a backward inferential
connection to a concept $B$, $(A \overrightarrow{\neg} B)$, for an agent $s$ if the following condition holds:

If $s$ has an interest in applying $A$ to $o$, then $s$ will adopt an interest in applying $B$ to $o$ for the purpose of believing $Ao$ on the basis of $Bo$.

Inferential connections also need to account for reason strengths. The strength of the inferential connection between two concepts will be the degree to which the application of one concept is a reason for the application of the other. Suppose, Mary is one hundred percent confident that an object, $o$, is a bird and suppose her concept BIRD has the forward inferential connection BIRD $\overrightarrow{\neg}$ FLIES. From $[\text{BIRD}o]$, Mary can conclude $[\text{FLIES}o]$. However, Mary likely will not be as confident in her conclusion that $o$ flies as she was that $o$ is a bird. This is because not all birds fly. The strength of BIRD $\overrightarrow{\neg}$ FLIES is meant to capture this idea. Precisely what is captured by the strength of an inferential connection will depend to some degree on the theory of defeasible reasoning we adopt. For example, a Bayesian might interpret “strength of $A \overrightarrow{\neg} B$” as “probability of $o$ given $Ao$.” For backwards inferential connections, “strength of $A \overleftarrow{\neg} B$” would be interpreted as “probability of $Ao$ given $o$.” Since I wish to remain neutral between theories of defeasible reasoning, I won’t commit to a specific interpretation here.

The strength of inferential connections is important for compositionality because as we compose concepts we will have conflicting inferential connections. In these cases the stronger inferential connections will beat out the weaker.

Although we have talked as though composing WATERMELON and ART results in WATERMELON-ART, in fact, there are a variety of ways to compose these two concepts. For example, WATERMELON-OR-ART would be a different composition of the same concepts. These different methods of composing concepts are what I will call *compositional functions*. One might initially be inclined to think that WATERMELON-OR-ART has three concepts, WATERMELON, OR, and ART, as constituents. I will address that topic in Section 5. For now I will simply stipulate that the concept OR is not a constituent of compositional concepts such as WATERMELON-OR-ART. Rather, WATERMELON-OR-ART is the result of applying the compositional function for “or” to the concepts WATERMELON and ART.
4.1 Compositional functions of logical operators

Now you almost certainly want to skip to section 5. I’d love comments on the rest but it gets quite hairy.

For the remainder of this section let’s consider two concepts $A$ and $B$ with the following forwards and backwards inferential connections:

\[ A = \{ A \rightarrow \circ R_1, \ldots, A \rightarrow \circ R_m, A \leftarrow \circ P_1, \ldots, A \leftarrow \circ P_k \} \]
\[ B = \{ B \rightarrow \circ S_1, \ldots, B \rightarrow \circ S_n, B \leftarrow \circ Q_1, \ldots, B \leftarrow \circ Q_l \}. \]

4.1.1 Conjunction

Likely, the most commonly used compositional function is conjunction. Let $\oplus \land (A, B)$ be the composition of $A$ and $B$ via conjunction. So we would represent BROWN-COW as $\oplus \land (\text{BROWN, COW})$. Notice first that $\oplus \land$ is not just the logical operator $\land$. As a logical operator $\land$ takes propositions, sentences, or other truth evaluable entities as arguments (e.g. “It is raining $\land$ The ground is wet”). On the other hand, $\oplus \land$ takes concepts as arguments and concepts, in and of themselves, do not have truth values. However, as we will see in a moment, the compositional function $\oplus \land$ is tightly connected to the logical operator $\land$.

To form the concept $\oplus \land (A, B)$ the agent must formulate a new set of forwards and backwards inferential connections. We will look at each in turn. For the forwards connections the agent begins by supposing that some non-specific $o$ is such that $A o$ and $o$. $A o$ is a reason for $R_1 o, \ldots, R_m o$ and $o$ is a reason for $S_1 o, \ldots, S_n o$. Since nothing else has been supposed of $o$, the initial degree of justification for $R_1 o, \ldots, R_m o, S_1 o, \ldots, S_n o$ will be based only on the strength of the inferential connections $A \rightarrow \circ R_1, \ldots, A \rightarrow \circ R_m$ and $B \rightarrow \circ S_1, \ldots, B \rightarrow \circ S_n$.

Let $B_i$ be the set of the beliefs the agent holds on the supposition $A o$ and $o$ (other than $A o$ and $o$ themselves) at the $i$th step of the reasoning process. Initially,

\[ B_1 = \{ R_1 (o), \ldots, R_m (o), S_1 (o), \ldots, S_n (o) \}. \]

The agent then adopts an interest in defeaters for each member of $B_1$. The agent also adopts an interest in any interesting conclusions about $o$ that can be derived from $B_1$.\footnote{In many cases an agent will be creating a compositional concept in the furtherance of some other goal. For instance, if an agent hears “grey squirrels harvest gold,” then in forming the compositional concept GREY-}
members of $B_1$ stay, which go, and which new beliefs are added. At each stage of reasoning the elements of $B_i$ may be changed to generate $B_{i+1}$. New conclusions will be drawn, new defeaters found or existing defeaters defeated themselves.

At any step of the reasoning process the agent may form the forward inferential connections for $\oplus\land(A, B)$. $B_i$ will always consist of a set of (so far) undefeated beliefs based on the supposition of $Ao$ and $o$. For each belief $U_j o \in B_i$ the agent forms a forward inferential connection $\oplus\land(A, B) \overrightarrow{o} U_j$, where $U_j$ is the concept tokened in $U_j o$. One result of this theory is that the inferential connections for most concepts are never entirely secure. More reasoning could always be done and this reasoning might call old connections into doubt or add new connections.

Backwards inferential connections for $\oplus\land(A, B)$ are formed somewhat differently. Rather than supposing there is an $o$ such that $Ao \land o$, the agent adopts a suppositional interest in $Ao \land o$. This will lead to an interest in $Ao$ and $o$ individually. Strictly speaking an agent could simply stop at this point and form the backwards connections. The result would be $\oplus\land(A, B) \overrightarrow{o} A$ and $\oplus\land(A, B) \overrightarrow{o} B$. However, there may be something to be gained by tracing the backwards inferential connections of $Ao$ and $o$ as well. In particular, it may turn out that for some $P_i$ and $Q_j$, which are connected to $A$ and $B$ respectively, that $P_i = Q_j$. In this case the agent will have special reason to adopt an interest in $P_i$ since that will get her both $A$ and $B$.

This gives the basic picture of how the compositional function $\oplus\land$ works. However, it overlooks an interesting problem. Consider the concept apple. The inferential role of apple partially consists of the inferential connections apple $\overrightarrow{o}$ red and apple $\overrightarrow{o}$ green, where the strength of the former is higher than that of the latter. Now suppose we want to form the compositional concept $\oplus\land$ (fresh, apple). According to the technique outlined above the agent forms a set of beliefs, $B_1$, on the supposition of fresh $o$ and apple $o$. Included in $B_1$ will be the beliefs red $o$ and green $o$. At some later stage of reasoning, $t$, $B_t$ would contain red $o$ but not green $o$. This is because the agent can generate an argument from red $o$ to $\neg$green $o$ and the strength of that argument will be stronger than the argument for green $o$ (since apple $\overrightarrow{o}$ red is stronger than apple $\overrightarrow{o}$ green). So the inferential squirrel the agent may adopt a special interest in conclusions that can be drawn about the grey-squirrels harvesting habits.
role of $\oplus \wedge \text{(FRESH,APPLE)}$ will contain the inferential connection $\oplus \wedge \text{(FRESH,APPLE)} \overrightarrow{o} \text{RED}$, but it will not contain $\oplus \wedge \text{(FRESH,APPLE)} \overrightarrow{o} \text{GREEN}$. But this surely is not right. Freshness doesn’t eliminate the possibility that an apple is green.

In IRS the content of a concept doesn’t contain the strongest inferential connections, it contains all inferential connections that are cognitively significant. APPLE has an inferential connection to both RED and GREEN because an agent ought to conclude that an apple is green if she learns it isn’t red. Of course, when we apply APPLE to a real object we have a reason to think it is red and therefore not green. But o isn’t a real object. The upshot is that the kind of reasoning used in forming a compositional concept has some constraints that normal reasoning does not.

The solution is to place a restriction on when one belief can defeat another during the formation of a compositional concept. Let’s call this *compositional-defeat*. Suppose $Uo$ is a defeater for $To$ and $Uo, To \in B_i$ at some stage of reasoning while forming $\oplus \wedge (A, B)$. We can define compositional defeat in the following way.

$Uo$ is a compositional-defeater for $To$ if,

1. there is at least one argument for $To$ which depends upon $Ao$, but does not depend upon $o$,
2. there is at least one argument for $Uo$ which depends upon $o$, but does not depend upon $Ao$, and
3. the strongest argument picked out by condition 1. is stronger than the strongest argument picked out by condition 2.

This handles the the example above because the argument for $\neg \text{RED} o$ and for $\text{GREEN} o$ both depend exclusively on $\text{APPLE} o$.

### 4.1.2 Disjunction

Disjunction will work somewhat different. Let $\oplus \vee$ be the compositional function for disjunction. Generating the compositional concept $\oplus \vee (A, B)$ begins much as it does for $\oplus \wedge (A, B)$. The agent first supposes there is an $o$ such that $(Ao \vee o)$. From this supposition the agent will be able to draw some conclusions. In particular, if $Ao$ is a reason for $R_i o$, $o$ is a reason for $S_j o$, and $R_i = S_j$, then $(Ao \vee o)$ is a reason for $R_i o$. The reason strength in this case
will depend upon which theory of reasoning turns out to be right, but plausibly it will be the weakest of the strongest argument from $A_o$ and the strongest argument from $o$.

At first it might appear that a problem for $\oplus \lor$ occurs, similar to the APPLE case above. Consider the concept $\oplus \lor$(APPLE, BANANA) and suppose that apples are never yellow and bananas are never red. APPLE $o$ is a reason for RED $o$ and for GREEN $o$ and BANANA $o$ is a reason for YELLOW $o$ and GREEN $o$. Both APPLE and BANANA have an inferential connection to GREEN. So according to the prescriptions above, the agent should have an inferential connection from $\oplus \lor$(APPLE, BANANA) to GREEN. But if it is an apple it probably isn’t green (because it is most likely red) and if it is a banana it probably isn’t green (because it is most likely yellow). So while we can’t conclude what color it is, it seems that we shouldn’t conclude that an (APPLE $\lor$ BANANA) is green.

In fact, this is no different from the case of the fresh apple. That something is an apple or banana gives us some reason for thinking it is green in the same way that its just being an apple would give us a reason for thinking it is green. There are better reasons to think that an apple is red than green, but the reasons for red don’t eliminate the reasons for green, they only defeat them.

It also should be noted that there will be a connection from $\oplus \lor$(APPLE, BANANA) to $\oplus \lor$(RED, YELLOW) which will be stronger than the connection to GREEN. Since $\oplus \lor$(RED, YELLOW) defeats GREEN, it is still possible to defeat the inference in general. That is, you have no better reason to think it is GREEN than you did before (though you also have no particular reason to think it is RED or YELLOW either).

The backward inferential connections for $\oplus \lor(A, B)$ will work differently. Suppose $A$ and $B$ have inferential connections $\{A \leftarrow P_1, \ldots, A \leftarrow P_m\}$ and $\{B \leftarrow Q_1, \ldots, B \leftarrow Q_n\}$ respectively. Since the agent can infer $\oplus \lor(A, B)$ from either $A$ or $B$, any inferential connections to $A$ or $B$ will be an inferential connection to $\oplus \lor(A, B)$.

4.1.3 Negation

Negation is a bit more tricky than conjunction or disjunction. Consider the compositional function $\oplus \neg(A)$ expressed in English as “NON-$A$.” As above, suppose $A$ has the set of inferential connections, $A = \{A \leftarrow R_1, \ldots, A \leftarrow R_m, A \leftarrow P_1, \ldots, A \leftarrow P_k\}$
As a first attempt to explain how $\oplus \neg$ works we might try what we did with $\oplus \land$ and $\oplus \lor$. To get NON-A an agent might first suppose $\neg Ao$. Since $\{P_1 o, \ldots, P_k o\}$ are reasons for $Ao$, a defeasible version of modus tollens suggests that $\neg Ao$ is a reason against $\{P_1 o, \ldots, P_k o\}$. So the agent can infer $\{\neg P_1 o, \ldots, \neg P_k o\}$. The agent might then look for defeaters or *interesting* conclusions from this set of suppositional beliefs. At some point she uses these suppositional beliefs to form the inferential connections for $\oplus \neg(A)$. The result would look something like the following.

$$\{\oplus \neg(A) \quad \neg P_1, \ldots, \oplus \neg(A) \quad \neg P_k\}.$$ 

Now we have a bit of a problem though. Each $\neg P_i$ is, itself, the compositional concept $\oplus \neg(P_i)$. It is obvious that this is quickly going to lead to a regress problem. The same problem comes up for backwards inferential connections. Since the agent can infer $R_i o$ from $Ao$, she can infer $\neg Ao$ from $\neg R_i o$. So the backwards connections for $\oplus \neg(A)$ are

$$\{\neg R_1 \quad \oplus \neg(A), \ldots, \neg R_m \quad \oplus \neg(A)\}$$

In both the forward and backward directions the formation of one concept with $\oplus \neg$ requires forming more concepts with $\oplus \neg$. The concern here is not so much that it is computationally implausible—it only doubles the number of concepts an agent must maintain—but it does seem messy.

Fortunately there is no need for things to get messy. So far we have been treating forward and backward inferential connections in a purely positive sense. That is, $A \rightarrow B$ has been treated as meaning $Ao$ is a reason for $o$ and $A \leftarrow B$ has been treated as meaning $o$ is a reason for $Ao$. But really there are going to be just as many cases where $Ao$ is a reason against $o$ or vice-versa. For example, it is a fairly salient feature of penguins that they do not fly. There are two ways this might be represented in IRS. It could be that there is an inferential connection from PENGUIN to NON-FLYING. Or it could be that there is an inferential connection from PENGUIN to FLIES where the connection itself indicates that it is a reason against – rather than a reason for – flying. The latter of these possibilities seems

\[^3\text{In fact, the agent would not look for defeaters for much the same reason that in applying } \oplus \land \text{ an agent cannot allow one inference from } A \text{ to defeat another inference from } A.\]
the more useful and efficient. Rather than forming a negated concept \(\neg o\) every time \(Ao\) is a reason against \(o\), this is just stored in the weights between \(Ao\) and \(o\).

By taking care of negation in the inferential connections we free the agent from the regress problem mentioned above. Forming \(\oplus\neg(A)\) does not require forming the compositional concepts \(\oplus\neg(P_i)\).

5 Concepts of compositional functions

I want to stave off a possible objection. The objection goes roughly like this, “I would have thought that WARM-AND-FUZZY was composed of three concepts, WARM, FUZZY, and AND. But the concept AND doesn’t seem to appear in your account. Instead there is a compositional function \(\oplus\land\). Furthermore, the very fact that I can consider that ‘and’ is not a concept but a function indicates that I have a concept of ‘and.’ After all, I know what you are talking about.”

There is, in fact, a concept AND and it is pretty easy to capture it via inferential role. But AND is not a constituent of a concept like WARM-AND-FUZZY despite the English expression. The concept AND is a concept of the compositional function \(\oplus\land\). This is a sort of use/mention distinction for concepts. It is possible to think about many mental objects (processes, representations, etc). When we think about one of these mental objects we are tokening a concept of the object. But we do not need to token a concept of a mental process or mental representation in order to use the process or representation. For example, CONCEPT is itself a concept. But when when BIRD is employed it isn’t necessary that CONCEPT be tokened as well. Compositional functions are no different.

I think this has led to some confusion as the logical operators are often flouted as the best examples for explanation by inferential role (Peacocke, 1992; Fodor, 1998, see). This leads one to think that each instance of “and” in an English expression is an opportunity to plug in the flagship example of IRS.
6 Other Compositional Functions

I believe I’ve explained how many (perhaps most) compositional concepts get their initial inferential role. This is because many compositional concepts are formed with the ⊕∧ and ⊕¬ functions. However, a number of compositional concepts have been left out of this explanation.

Consider the concept large-mouse. Initially we might think this is formed by composing large and mouse with the ⊕∧ function. Mouse is not problematic but large presents some difficulties. The first issue is that on its own large is not a concept that can be applied to mice. Mice are not large they are small. An initial solution would suggest that the word “large” stands for two concepts; One concept is large-simpliciter and the other is large-for-a- (a better solution is offered in footnote 4). A concept like large-for-a- would allow us to distinguish large mice from normal mice without having to say that mice can be large simpliciter. Unfortunately, it is pretty clear that large-for-a- cannot, itself, be a concept. Rather it must be a compositional function. An example will help illustrate. Suppose I see Jerry, a rather large mouse, for the first time. I want to apply large-for-a-mouse to Jerry. If large-for-a- were a concept, then I would compose large-for-a- and mouse. Furthermore if large-for-a- were a concept, then it should also make sense for me to apply it to Jerry without first composing it with mouse. However, large-for-a-jerry is nonsense. In fact, for any object, o, large-for-a-o is nonsense. Large-for-a- cannot be applied to objects. Rather it is applied to concepts in the same way that a function is applied to a domain. In the present case, what is needed is to apply the function ⊕large-for-a to mouse and get large-for-a-mouse. We can now apply the new concept to Jerry.

That there is a compositional function ⊕large-for-a is not a particular problem until we consider that there will be lots of these kinds of compositional functions. The concern is that it will often turn out that what we thought was a composition of two concepts is actually a compositional-function applied to a concept. The more of these functions there are, the more ad hoc the theory appears.

The example above makes it easy to see how more such examples could be generated. Active-three-toed-sloth, fast-snail, and loud-whisper are all compositional concepts which cannot be formed using the compositional functions mentioned thus far. If the present theory required a unique compositional function for each of these concepts, then it
would appear quite *ad hoc*. Thankfully I believe there is a way to work around this.

Many apparent compositional functions are themselves composed of simpler compositional functions and concepts. For example, $\oplus_{\text{large-for-a}}(C)$ can be readily broken down into several smaller parts, some of which are compositional functions and some of which are just concepts. The result would be something like the following: $\oplus_{\text{more}}(\text{size}, \oplus_{\text{typical}}(C))$. Here $\oplus_{\text{more}}$ and $\oplus_{\text{typical}}$ are compositional functions, while size is simply a concept.

The concept LARGE-MOUSE roughly translates as “a mouse with more size than a typical mouse.” The $\oplus_{\text{more}}$ would be a genuine compositional function, but it would be the basis of many apparent compositional functions such as $\oplus_{\text{old-for-a}}$, $\oplus_{\text{long-for-a}}$, etc.

There is one other compositional function which is worth discussing because of the number of compositional concepts it is used to produce. Consider the compositional concept RAIN-COAT. Clearly RAIN-COAT is not merely the conjunction of RAIN and COAT. Nor will any other logical operators do. A rain coat is a coat for rain. That is, it is a coat with a special purpose. The question is how ‘purpose’ enters into the concept. There are several possibilities here. First, it might be that the concept PURPOSE is one of several concepts which are composed to make RAIN-COAT. Second, it might be that there is a dedicated compositional function for composing concepts when one has a purpose relating to the other. Each of these options has some problems though.

\[\text{large-simpliciter} = \oplus_{\text{more}}(\text{size}, \oplus_{\text{typical}}(\text{object}))\]

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4 This also allows us to explain the concept LARGE-SIMPLICITER. LARGE-SIMPLICITER simply fills in object for C. What we end up with is that LARGE-SIMPLICITER = \( \oplus_{\text{more}}(\text{size}, \oplus_{\text{typical}}(\text{object})) \)
References


